



# How Can Motion be Described? and Explained?

## Lesson 14: Torque and the Stability of Structures

### Stable Structures



Melbourne Cricket Ground (MCG)



Eureka Tower Melbourne



West Gate Bridge

Explain why structures should be stable.

### What are the conditions for a structure to be stable?

Newton's Second Law,  $F_{net} = ma$ , gives us a clue.

A structure should have ..... initial velocity and zero .....

Hence net force on all parts of a structure must equal .....

A body is in **translational** ..... when the sum of the forces acting on it is equal to zero.

$$\Sigma F = 0$$

One **condition** for a stable structure is that it all of its component parts must be in translational equilibrium.

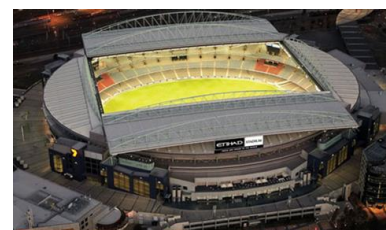
### Spanning Roof Structures



Rod Laver Arena



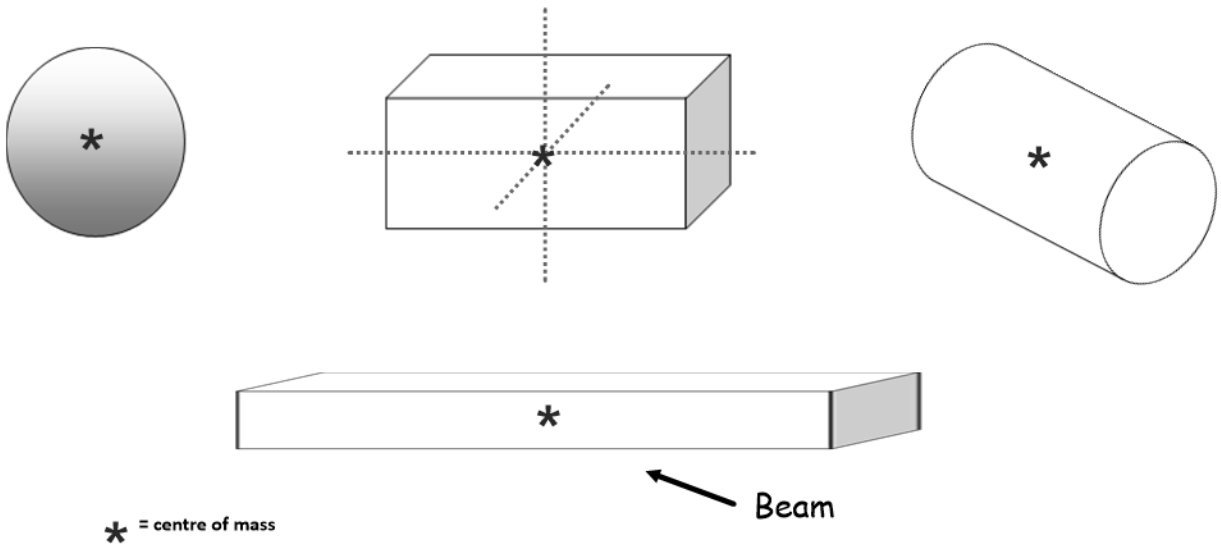
Wimbledon



Ethiad Stadium, Melbourne

### Centre of Mass

The point in a body where all the ..... can be considered to be concentrated.

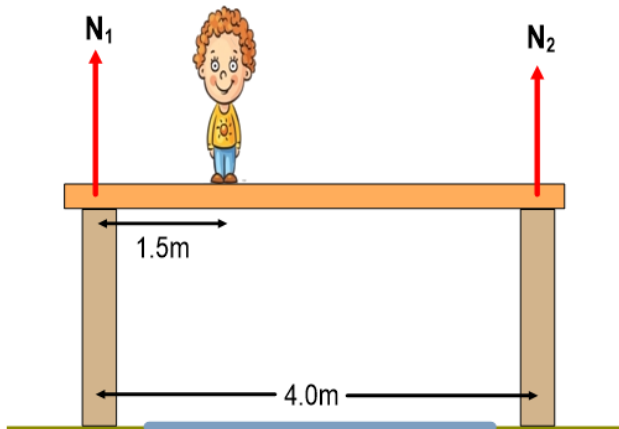


Centre of mass is a useful concept for analysing structures.

Weight forces ( $mg$ ) should always be drawn from the ..... of mass of a structural element.

### Worked Example

For the simple beam bridge shown below, find the size of the force marked  $N_1$  (in Newtons) if  $N_2 = 200\text{N}$ . Assume acceleration due to gravity,  $g = 10\text{ ms}^{-2}$ .



mass of child,  $m_c = 40\text{kg}$

mass of beam,  $m_b = 10\text{kg}$

### Solution

**Solution (continued)**

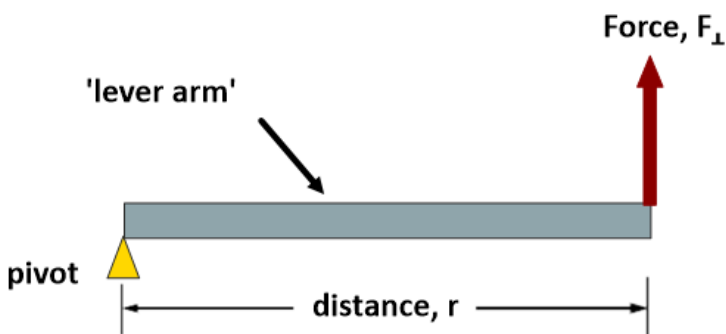
## Rotational Forces

### Torque

Torque is a ..... force that produces a twisting, turning or rotational effect.



### Torque Formula



The *torque* acting on a body (lever arm) is given by the formula

$$\tau = rF_{\perp}$$

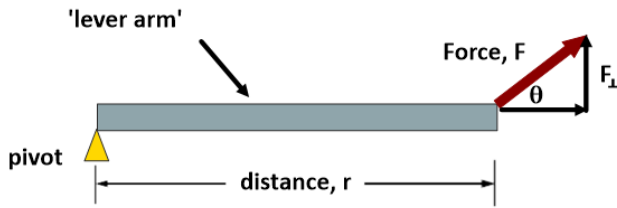
where  $\tau$  = ..... (Nm)

$r$  = distance from ..... to force (m)

$F_{\perp}$  = perpendicular component of force (N)

## Rotational Forces (Continued)

Calculating Torque when force,  $F$  is not ..... to the lever arm



$$\sin\theta = \frac{F_{\perp}}{F}$$

$$\text{so } F_{\perp} = F\sin\theta$$

$$\tau = rF\sin\theta$$

where  $\tau$  = torque (Nm)

$F$  = force (N)

$r$  = distance from pivot to force (m)

$\theta$  = angle between ..... arm and applied ..... (degrees)



**Torque** is at its ..... when force,  $F$  is perpendicular to the lever arm.

$$\text{so } F_{\perp} = F$$

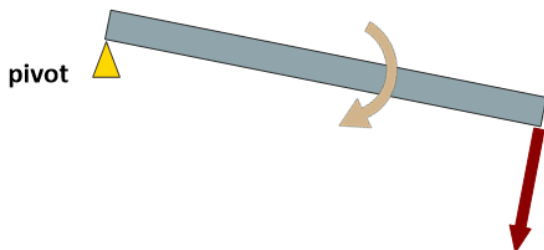


**Torque** is between maximum and zero



**Torque** is ..... when force,  $F$  is ..... to the lever arm.

### Indicating Direction of Torque



This is a ..... torque and is sometimes referred to as a **positive** torque.

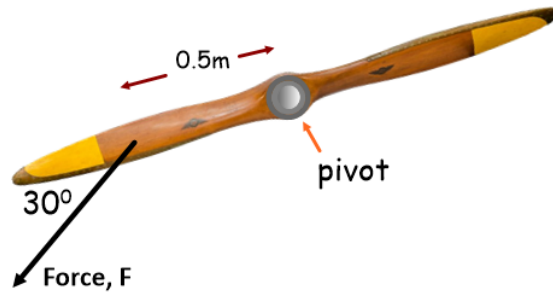
**Anticlockwise** torque



is sometimes referred to as a ..... torque.

## Worked Example

The engine of a vintage aeroplane is 'crank started' by applying a torque to the propeller blades (and then standing back quickly!)



- The pilot applies a force of 100N at an angle of  $30^\circ$  and a distance of 0.5 metre from the pivot, as shown above. Calculate the size and direction of the torque.
- How could the pilot apply a greater torque with less effort?

## Solution

## Rotational Equilibrium

A body must be in rotational equilibrium to be .....

A body is in **rotational equilibrium** when the sum of the ..... acting on it is equal to zero.

$$\Sigma \tau = 0$$

For a body or structure to be in **Static Equilibrium** (and hence be stable), it must be in both t..... equilibrium and r.....equilibrium.

$$\Sigma F = 0$$

and

$$\Sigma \tau = 0$$



Static Equilibrium

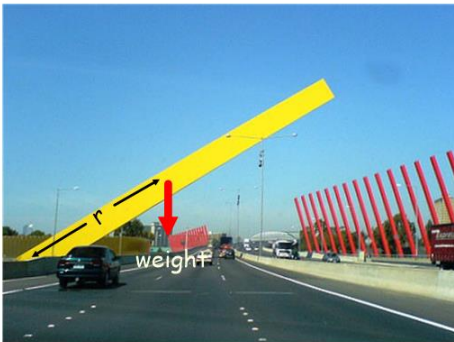


Not in Static Equilibrium



The Ship Worker (in static equilibrium)

## The 'Cheesetick'

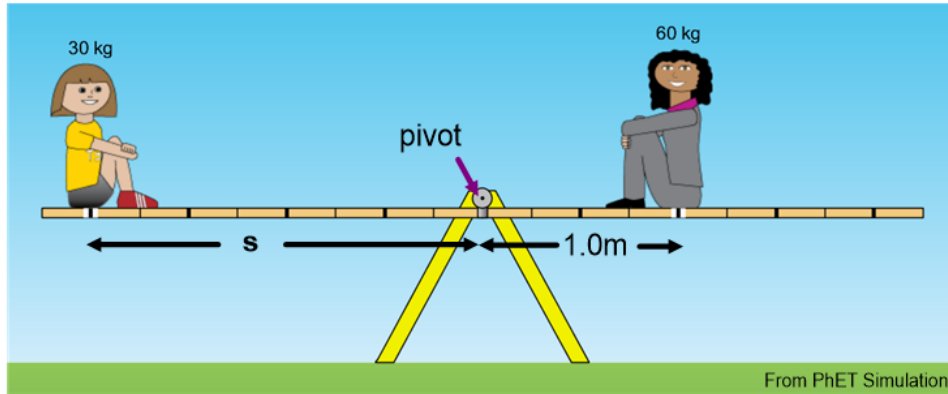


There is a huge ..... torque acting on this structure. The foundations would need to supply an equal **anticlockwise** torque to keep the 'cheesetick' stable.

Extra Notes

## Worked Example

An adult and a child are sitting on a see-saw which is in perfect 'balance' so that it is not moving.



- Draw a force diagram and explain why the 'structure' is in translational equilibrium.
- Determine the value of distance  $s$  which allows the structure to be in rotational equilibrium.
- Is this a stable structure?
- What would cause it to become unstable?

## Solution

**Solution (continued)**

## Simple Structural Elements

### Beams



Ancient temple in Egypt



Stonehenge in England (prehistoric)



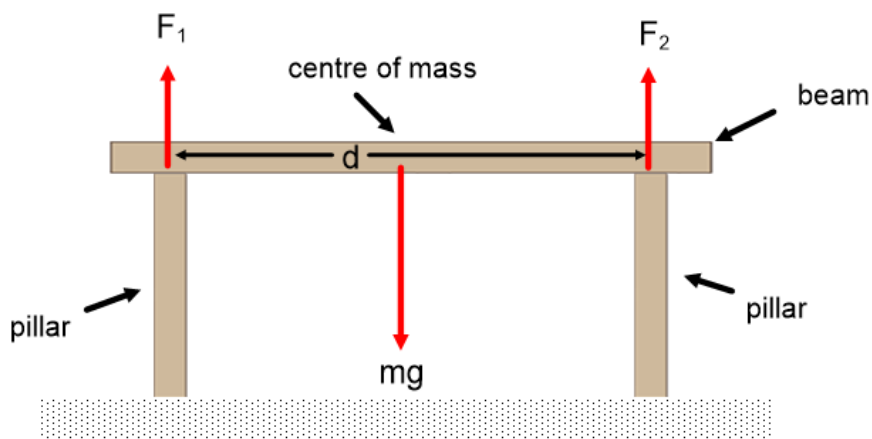
Parthenon

### Beam Bridges



Why can modern structures span far greater distances?

Beam Bridges can be analysed using the conditions for **translational** and **rotational** equilibrium.



**Translational Equilibrium**

$$\Sigma F = 0$$

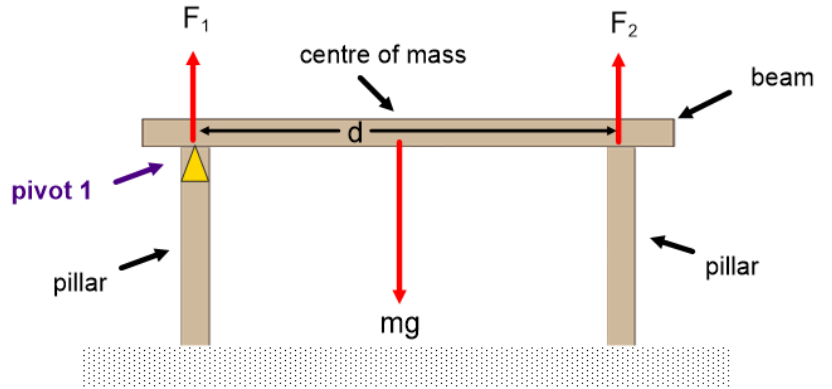
$$F_1 + F_2 - mg = 0$$

**Rotational Equilibrium**

$$\Sigma \tau = 0$$

Taking pivot 1 as the reference point,

$$d/2 \times \dots\dots\dots - d \times \dots\dots\dots = 0$$



**Principle of Moments**

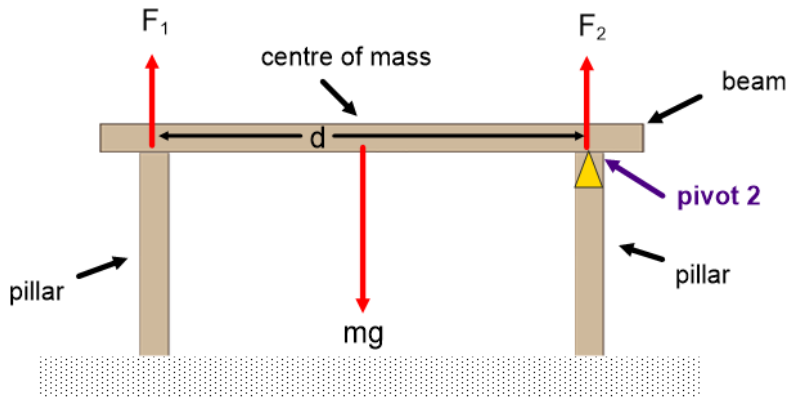
When analysing situations where more than one ..... acts on a body, the body will be in rotational equilibrium if the net torque about a ..... (or pivot) point equals zero.

**Rotational Equilibrium**

$$\Sigma \tau = 0$$

Taking pivot 2 as the reference point,

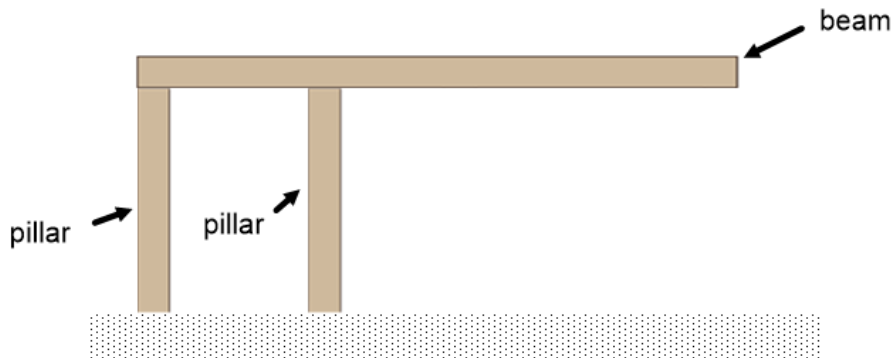
$$d \times F_1 - \dots\dots\dots \times mg = 0$$



**Simple Structural Elements (continued)**

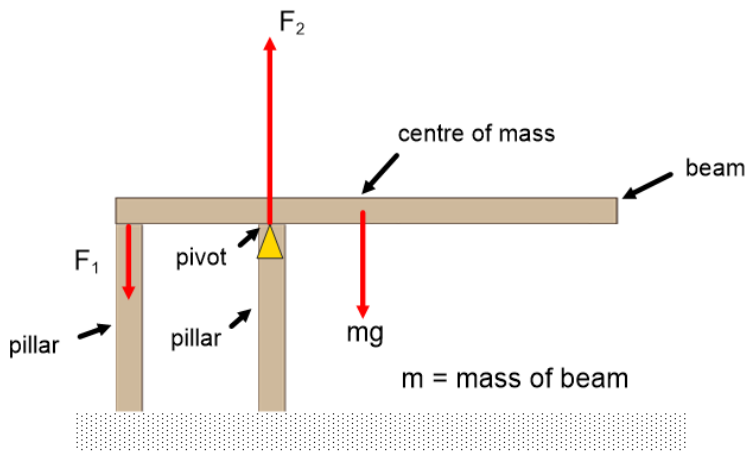
**Cantilever**

A beam that extends beyond its ..... structure.



### Analysing the cantilever

Draw in all forces acting on the beam as shown below.



**Translational Equilibrium**

$$\Sigma F = 0$$

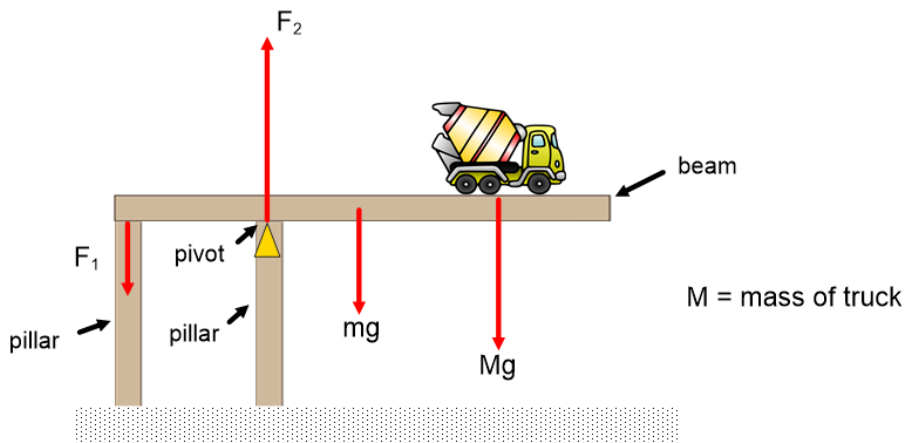
$$F_2 - \dots - mg = 0$$

**Rotational Equilibrium**

$$\Sigma \tau = 0$$

Take moments (measure torques) about ..... to determine any unknown quantities

### Cantilever with load

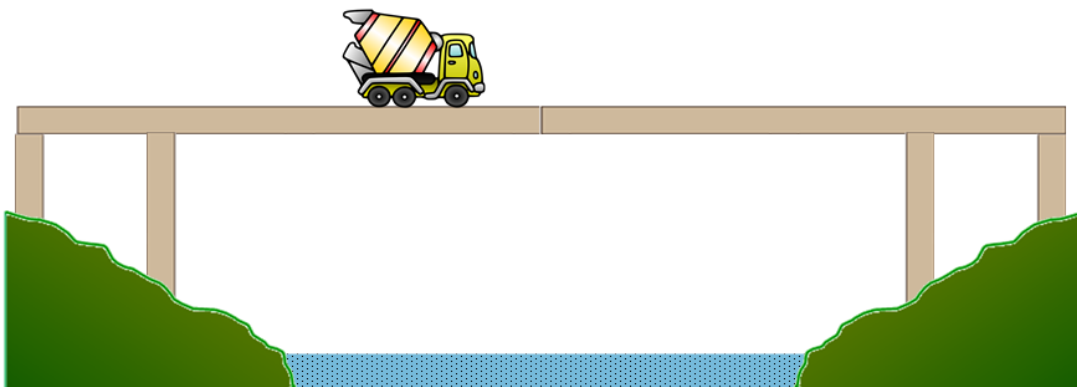


Translational Equilibrium  $\Sigma F = 0$   $F_2 - F_1 - mg - \dots = 0$

Rotational Equilibrium  $\Sigma \tau = 0$  etc

### Cantilever Bridge

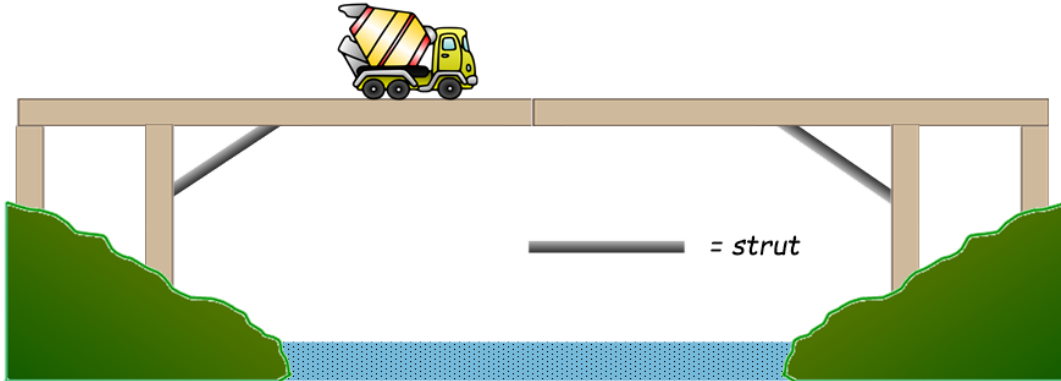
Just place two or more cantilvers together to make a bridge



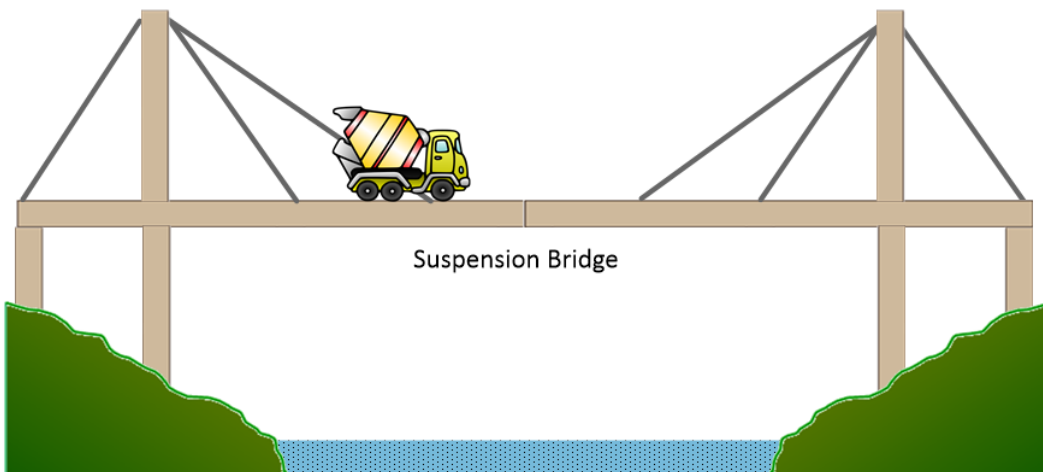
### Cantilever Bridges

Can be supported by ..... and/or .....

*Struts are rigid and always under .....*



Can be supported by **Ties** \_\_\_\_\_ *These are flexible and always under .....*



### Cantilever Bridges



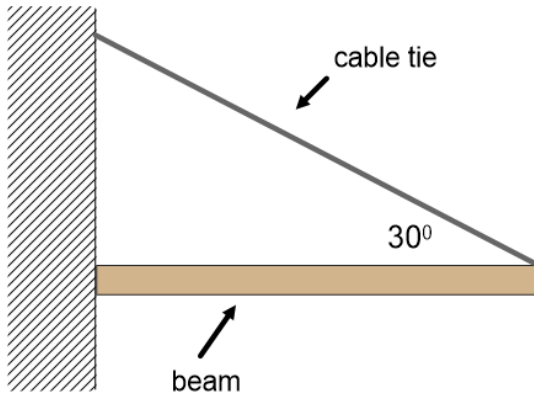
Portimao Suspension bridge, Portugal



Westgate Bridge, Melbourne

### Worked Example

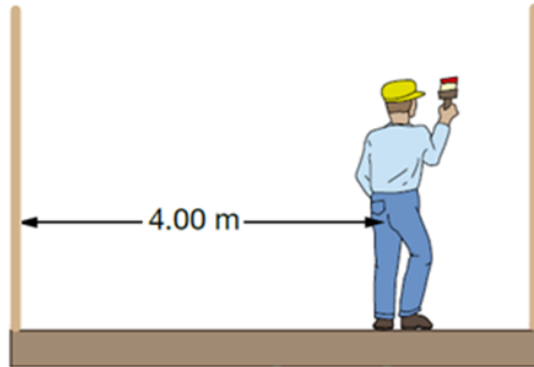
A uniform beam of mass 500 kg and length 20 m is supported by a cable tie as shown below  
Find the tension in the cable tie. (Take  $g = 10 \text{ ms}^{-2}$ )



### Solution (continued)

## Worked Example

While painting a tall building, a 70.0 kg painter stands 4.00 m from the end of a 6.00 m long plank that is supported by a rope at either end. The plank has a mass of 20.0 kg. Determine the tension in each rope.



## Solution (continued)

# Summary

## Stable Structures

**What are the conditions for a structure to be stable?**

Newton's Second Law,  $F_{\text{net}} = ma$ , gives us a clue.

A structure should have zero initial velocity and zero acceleration.

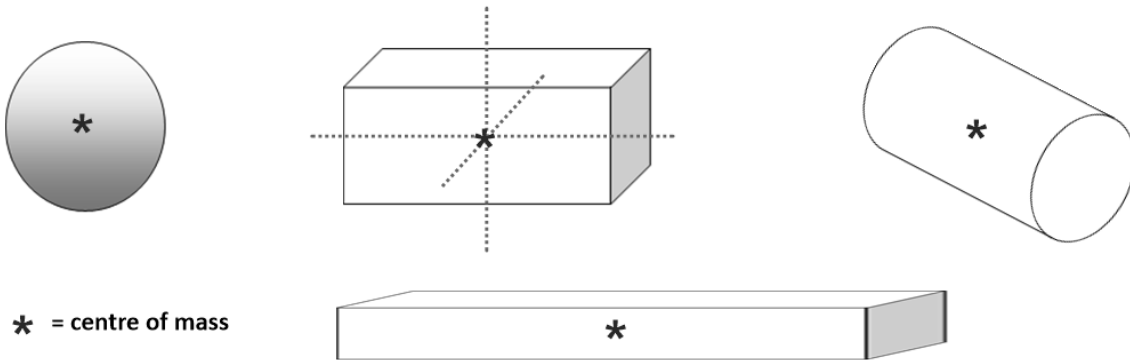
Hence net force on all parts of a structure must equal zero.

A body is in **translational equilibrium** when the sum of the forces acting on it is equal to zero.

$$\Sigma F = 0$$

## Centre of Mass

The point in a body where all the mass can be considered to be concentrated.



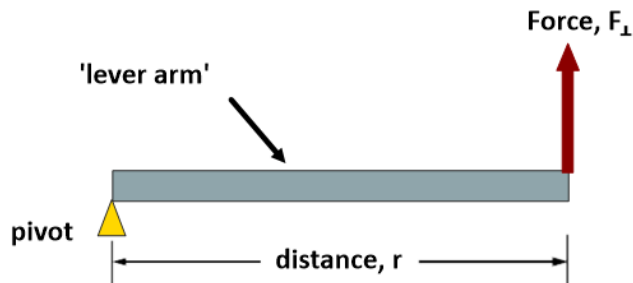
## Rotational Forces

### Torque

Torque is a rotational force that produces a twisting, turning or rotational effect.



### Torque Formula



The **torque** acting on a body (lever arm) is given by the formula

$$\tau = rF_{\perp}$$

where  $\tau$  = torque (Nm)

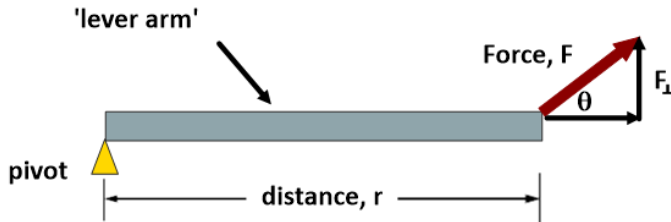
$r$  = distance from pivot to force (m)

$F_{\perp}$  = perpendicular component of force (N)

## Rotational Forces (Continued)

### Alternative Torque Formula

When the applied force is not perpendicular to the lever arm



$$\sin\theta = \frac{F_{\perp}}{F}$$

$$\text{so } F_{\perp} = F\sin\theta$$

$$\tau = rF\sin\theta$$

where  $\tau$  = torque (Nm)

$F$  = force (N)

$r$  = distance from pivot to force (m)

$\theta$  = angle between lever arm and applied force (degrees)



*Torque is at its greatest when force,  $F$  is perpendicular to the lever arm.*

$$\text{so } F_{\perp} = F$$



*Torque is zero when force,  $F$  is parallel to the lever arm.*

*A clockwise torque is referred to as a **positive** torque.*

*An anticlockwise torque is referred to as a **negative** torque.*

### Rotational Equilibrium

**A body must be in rotational equilibrium to be stable**

A body is in **rotational equilibrium** when the sum of the **torques** acting on it is equal to zero.

$$\Sigma \tau = 0$$

### Static Equilibrium

For a body or structure to be in **Static Equilibrium** (and hence be stable), it must be in both translational equilibrium and rotational equilibrium.

$$\Sigma F = 0$$

and

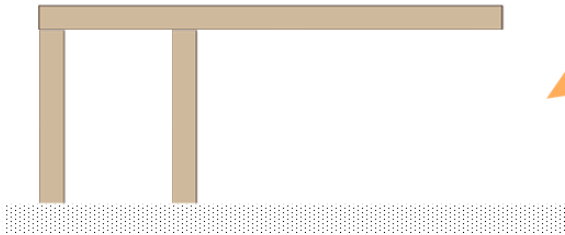
$$\Sigma \tau = 0$$

## Simple Structural Elements

### Beams - Beam Bridges



### Cantilever - Cantilever Bridges

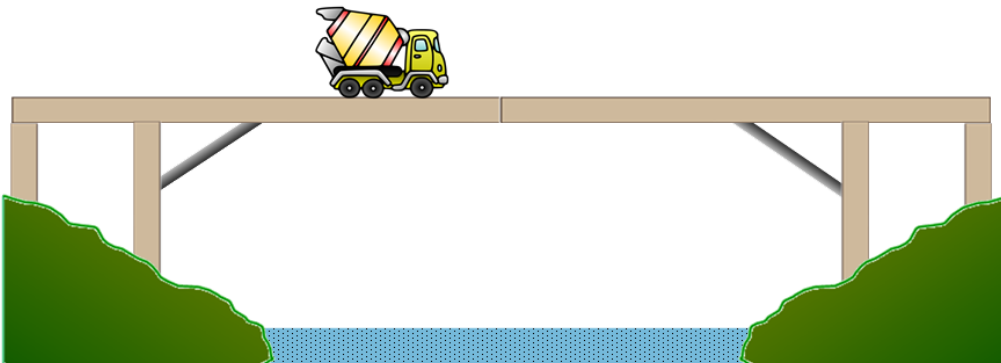



Translational Equilibrium  $\Sigma F = 0$  applies

Rotational Equilibrium  $\Sigma \tau = 0$  applies

### Cantilever Bridges

Can be supported by **Struts**  Rigid and always under compression



Cantilever Bridges Can be supported by **Ties** 

Flexible and always under tension

