How fast can things go?

Lesson 6: Einstein's Theory of Special Relativity

From the previous lesson…

Einstein's postulates for the theory of special relativity

I The laws of physics are the same in all inertial frames of reference.

II The speed of light in free space has the same value c in all inertial frames of reference.

Einstein's Gendanken (Thought) Experiments

Einstein was a theoretical physicist and did not undertake any practical experiments.

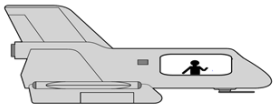
All of his experiments were 'thought' experiments or 'gendanken' experiments.

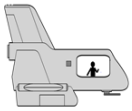
gendanken is a german word meaning thought

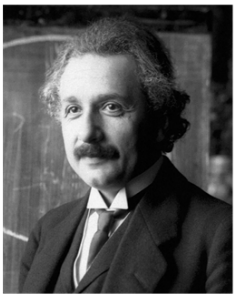
His findings have since been verified by extensive experimentation.

Implications of Special Relativity

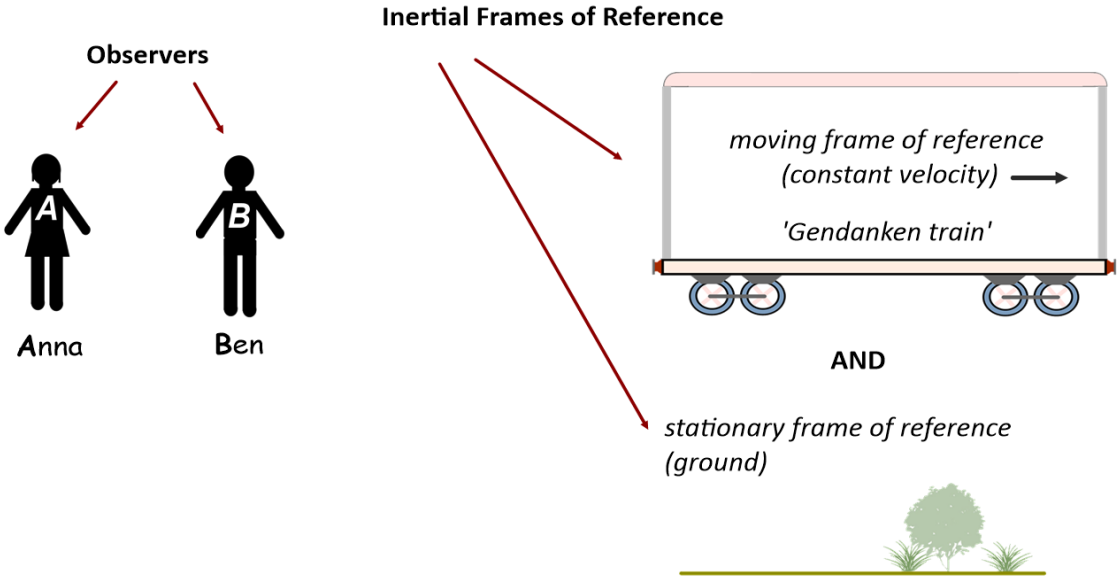
Einstein discovered that in order for all observers to measure the same velocity for a beam of light, time would have to .....' differently for different observers, and space would sometimes have to .....



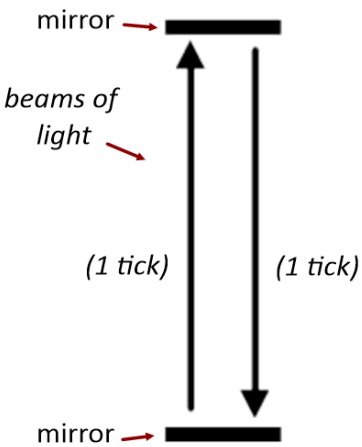




Thought Experiments relating to time dilation



Gendanken Light Clock.

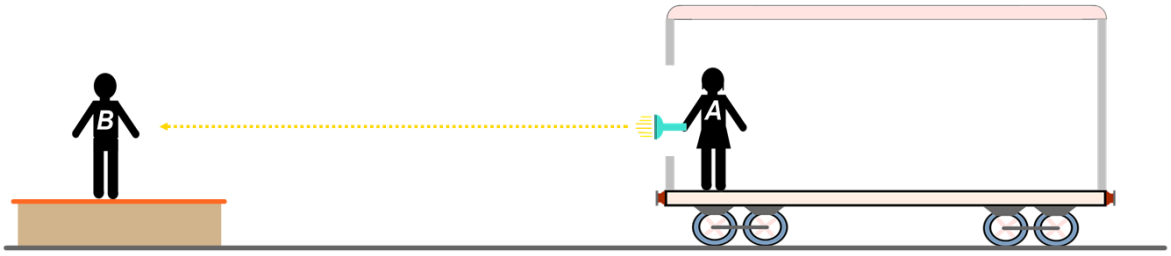


Two mirrors reflecting a beam of light back and forth.

Distance between mirrors = L

The journey from one mirror to another (up or down) counts as **one** .....of the clock

Experiment 1



v = 0

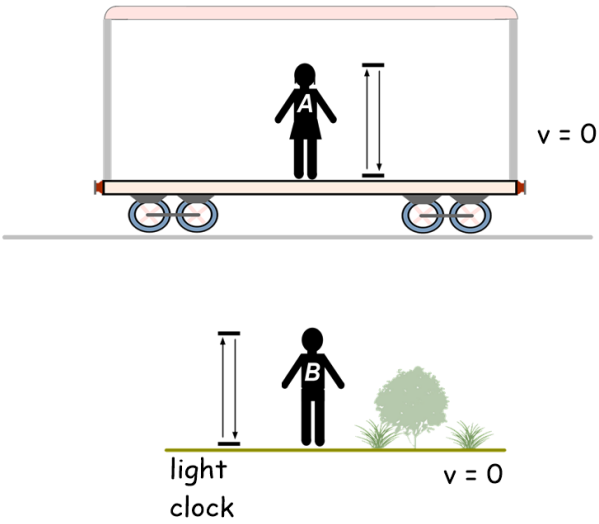
constant velocity, v = 25 ms^{-1}

Ben measures speed of light to be **c** = ... **ms^{-1}**

not c + v !

Anna measures speed of light to be c = 3.0 x 10^8 ms^{-1}

Experiment 2



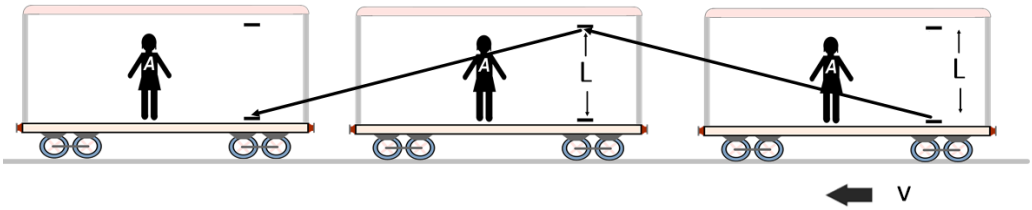
Write your own notes about this thought experiment.

Conclusion

All ..... and ..... perfectly synchronised

Experiment 3

railroad car moving to left with velocity, v



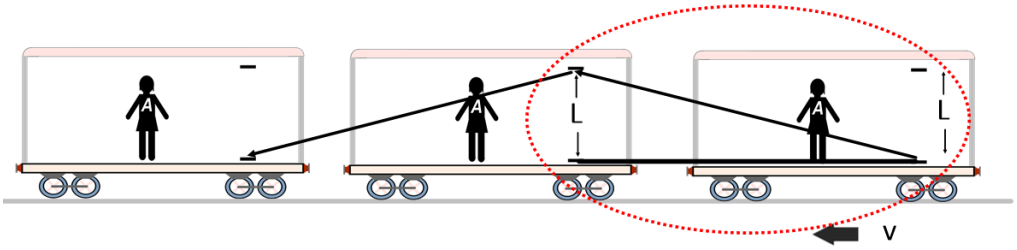
Ben sees Anna's watch slow down along with her clock

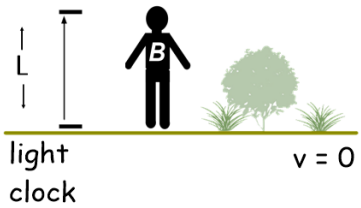
Write your own notes about this thought experiment.

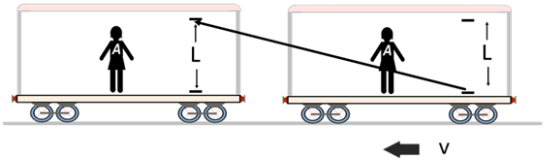
Conclusion

If someone moves in a straight line at constant velocity v with respect to you, then you will observe time passing more ……. for them.

Can we calculate how much time slows down?







distance = rate (velocity) x time

$$L = c\,t_o$$

Let

L = height of the light clocks

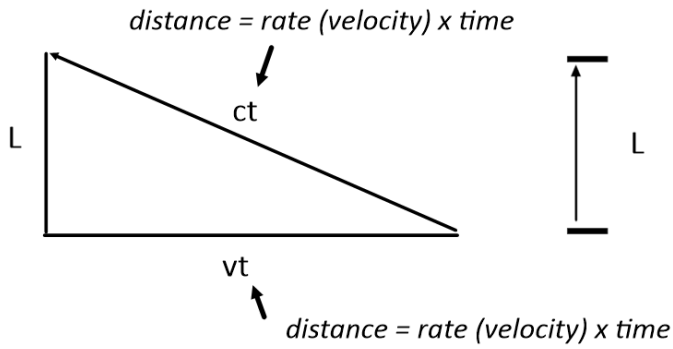
v = Anna's velocity

t_o = time between ticks on Ben's clock

t = time between ticks on Anna's clock, as Ben observes it.

c = speed of light

distance = rate (velocity) x time



$$(ct)^2 = (vt)^2 + L^2$$

but $L = c\,t_o$

hence $(ct)^2 = (vt)^2 + (ct_o)^2$

$$(ct)^2 = (vt)^2 + (ct_o)^2$$

$$c^2t^2 = v^2t^2 + c^2t_o^2$$

$$c^2t^2 - v^2t^2 = c^2t_o^2$$

divide both sides by c^2

$$\frac{\cancel{c^2}t^2}{\cancel{c^2}} - \frac{v^2t^2}{c^2} = \frac{\cancel{c^2}t_o^2}{\cancel{c^2}}$$

$$t^2 - \frac{v^2t^2}{c^2} = t_o^2$$

$$\left(1 - \frac{v^2}{c^2}\right) t^2 = t_o^2$$

hence

$$t^2 = \frac{t_o^2}{\left(1 - \frac{v^2}{c^2}\right)}$$

$$\boxed{t = \frac{t_o}{\sqrt{\left(1 - \frac{v^2}{c^2}\right)}}}$$

So if t_o is the time between ticks on Ben's watch, then he will observe a ..... interval of

$$t = \frac{t_o}{\sqrt{\left(1 - \frac{v^2}{c^2}\right)}}$$ between ticks on Anna's watch.

Note that this difference is not very much unless one is travelling at an extremely ..... velocity.

Einstein's Time Dilation Equation

$$t = \frac{t_o}{\sqrt{\left(1 - \frac{v^2}{c^2}\right)}}$$

Where t = the time as seen from the stationary frame of reference

t_o = the time in the moving frame

t_o is known as the proper time

The factor by which time is slowed down, $\frac{1}{\sqrt{\left(1 - \frac{v^2}{c^2}\right)}}$ is known as the ..... **factor,γ**

Or expressed more simply,

$$\boxed{t = \gamma t_0}$$

where $\gamma = \frac{1}{\sqrt{\left(1 - \frac{v^2}{c^2}\right)}}$

Where t = the time as seen from the stationary frame of reference

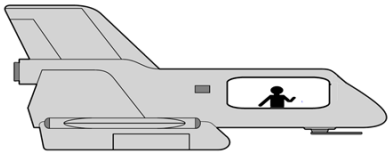
t_o = the time in the moving frame (proper time)

γ = Lorentz factor

Worked Example

Sam, an astronaut floating freely in space, observes a futuristic spaceship travelling past at a constant speed of 2.6×10^8 ms^{-1}. (Speed of light, c = 3.0×10^8 ms^{-1})

a. Calculate the Lorentz factor for this situation.

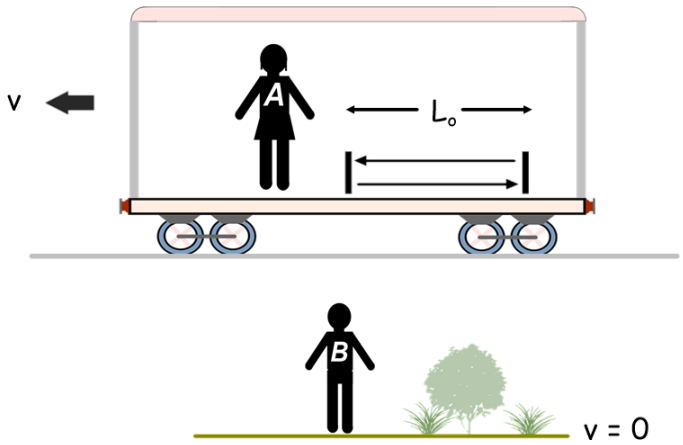


b. By what factor will time inside the spaceship slow down according to Sam's frame of reference.

Solution

Experiment 4 (Length contraction)

Anna's light clock of length L_o lies on its side as the railway car moves to the left with velocity v.

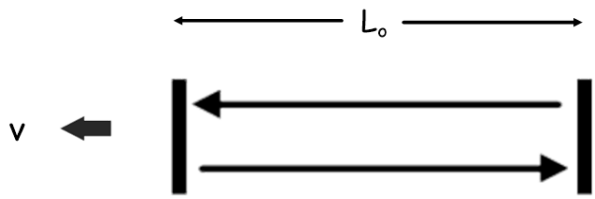


Ben observes the light clock from a stationary frame of reference. How will its length be affected?

L_o = length of light clock in Anna's frame of reference, (known as proper length).

L = length of light clock, as observed by Ben

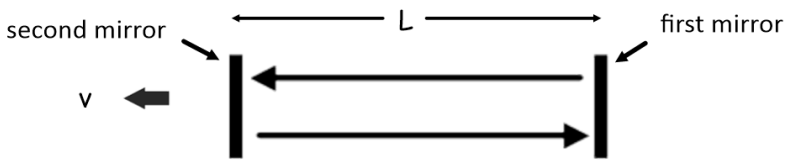
v = velocity of light clock.



We know that the time, t between ticks on the moving clock as Ben sees it, will be

$$t = \frac{t_o}{\sqrt{(1 - \frac{v^2}{c^2})}}$$

The light clock **as seen by Ben**



Let

t_1 = time that light takes light to go from the first mirror to the second mirror, as Ben perceives it.

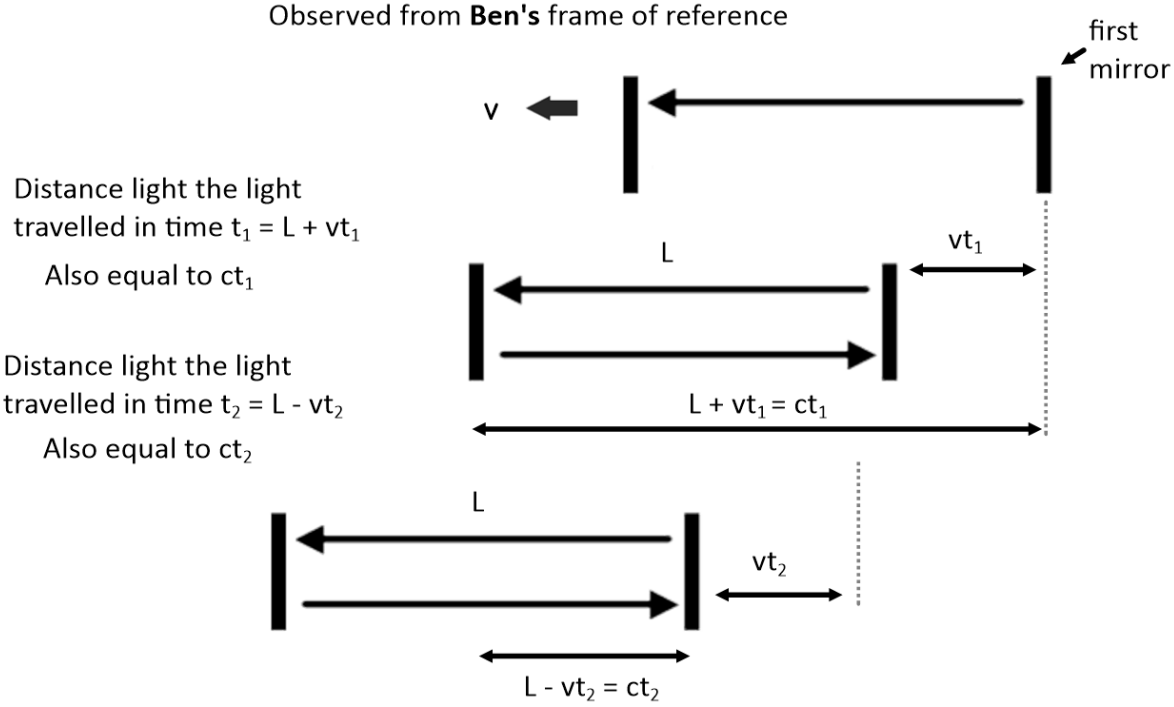
t_2 = time that light takes light to go from the second mirror back to the first, as Ben perceives it.

L = length of light clock, as observed by Ben

Next we will analyse the situation as the light clock moves to the left.

Experiment 4 (Derivation of length contraction)

Observed from **Ben's** frame of reference



$L + vt_1 = ct_1$

so $L = ct_1 - vt_1$

and $L = (c - v)t_1$

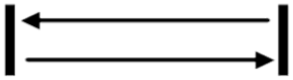
$$t_1 = \frac{L}{(c - v)}$$

$L - vt_2 = ct_2$

so $L = ct_2 + vt_2$

and $L = (c + v)t_2$

$$t_2 = \frac{L}{(c + v)}$$



$t_1 + t_2 = 2t$ As observed by Ben

$$2t = \frac{L}{(c - v)} + \frac{L}{(c + v)}$$

Experiment 4 (Derivation of length contraction)

$$2t = \frac{L}{(c - v)} + \frac{L}{(c + v)}$$

$$2t = \frac{L(c + v) + L(c - v)}{(c - v)(c + v)}$$

$$2t = \frac{Lc + Lv + Lc - Lv}{(c^2 - v^2)}$$

$$2t = \frac{2Lc}{(c^2 - v^2)}$$

$$t = \frac{Lc}{(c^2 - v^2)} = \frac{t_o}{\sqrt{(1 - \frac{v^2}{c^2})}}$$

but $t_o = \frac{L_o}{c}$

$$\text{so} \quad \frac{Lc}{(c^2 - v^2)} = \frac{L_o}{c\sqrt{(1 - \frac{v^2}{c^2})}}$$

and finally

$$L = L_o\sqrt{(1 - \frac{v^2}{c^2})}$$

$$L = L_o\sqrt{(1 - \frac{v^2}{c^2})}$$

where L = length of light clock, as observed by Ben

L_o = length of light clock in Anna's frame of reference.

v = velocity of light clock.

*L_o is known as the **proper length***

Conclusion

Since the factor $\sqrt{(1 - \frac{v^2}{c^2})}$ is less than 1, this shows that when an object is moving in a straight line with a fixed velocity v, a stationary observer (Ben) will see a contraction in its length.

The ..... an object travels, the ..... it appears.

Einstein's Length Contraction Equation

$$\boxed{L = L_o\sqrt{(1 - \frac{v^2}{c^2})}} \quad \text{or} \quad \boxed{L = \frac{L_o}{\gamma}} \quad \text{where } \gamma = \frac{1}{\sqrt{(1 - \frac{v^2}{c^2})}}$$

where L = contracted length as seen from stationary frame of reference

L_o = proper length (as seen in the moving frame of reference)

v = velocity of fast moving object

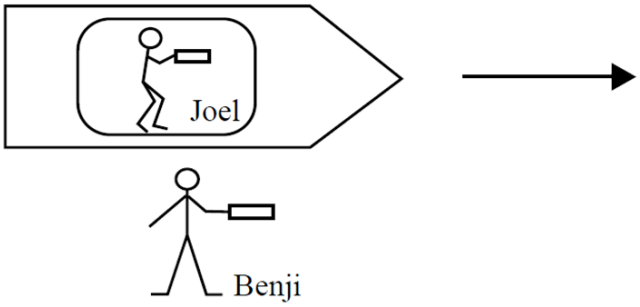
c = speed of light ($3.0 \times 10^8 ms^{-1}$)

γ = Lorentz factor

Worked Example

Astronaut Benji is floating freely in space when Joel's spaceship passes him at high speed. Benji is holding a ruler, and can see through the window of the spaceship that Joel is also holding a ruler. The two rulers are parallel to the direction of motion of the spaceship.

Speed of light, $c = 3.0 \times 10^8\ ms^{-1}$



Both Benji and Joel measure the length of their own ruler to be 1.0 m. Each can see the other ruler. Benji observes Joel's ruler to be 0.6 m long.

a. What is the proper length of Joel's ruler?

b. What is the length of Benji's ruler as observed by Joel?

c. What is the speed of Joel's spaceship as measured by Benji?

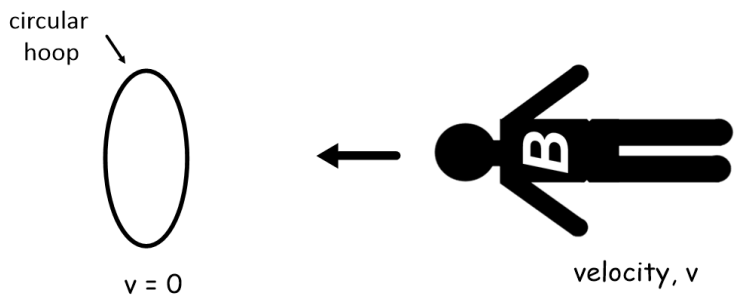
Solution

Solution

Experiment 5

What happens to lengths that are perpendicular to the direction of travel?

We'll begin by **assuming** that the moving object contracts.



If Ben moves relative to the hoop then (if our assumption is correct) he will contract and hence **he will pass through the loop.**

If the hoop moves relative to the Ben then (if our assumption is correct) the hoop will contract and hence **Ben will not pass through the loop.**

This is a contradictiom, so the original assumptiomn must be **incorrect.**

Conclusion

The only lengths that change are those that are **parallel** to our line of motion.

Proper time and length

These are the time and length as measured by an observer who is in the **same** frame of reference.

Proper time is the time between two events which occur at the ..... point in space.

Proper length is the distance between two points as measured by an observer who is **at** ..... with respect to these two points.

The Lorentz Factor

$$\gamma = \frac{1}{\sqrt{(1 - \frac{v^2}{c^2})}}$$

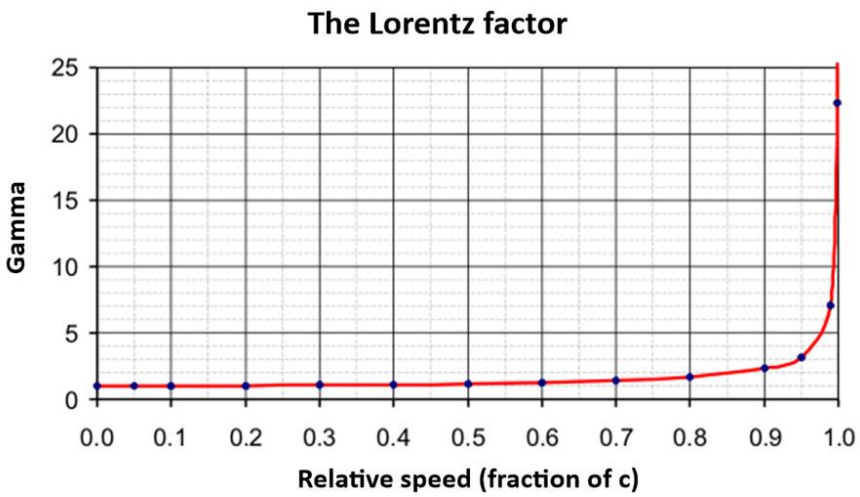
Denominator $\sqrt{(1 - \frac{v^2}{c^2})}$ is always $\leqslant 1$

so γ is always greater than or equal to 1

Time dilation and length contraction are really only noticeable at speeds greater than ....., (approximately 1.5×10^8 ms^{-1}).

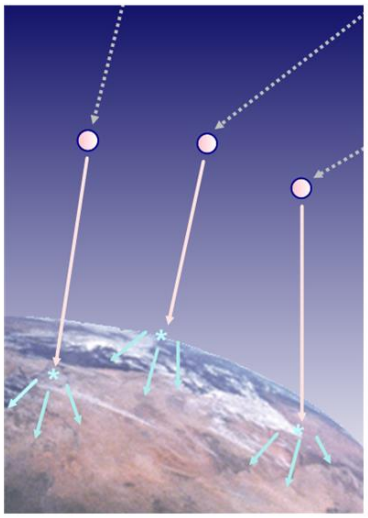
Lorentz factor at various speeds

v/c (%)	γ
0	1
1	1.00005
10	1.0005
50	1.155
86.6	2.0
90	2.3
99	7.1
99.9	22.4



Experimental Evidence

Muons



Muons are subatomic particles belonging to the lepton family.

Created by collisions of of cosmic rays with atoms in our upper atmosphere.

Muon speed is about 0.995c which is almost equal to the speed of light.

A muon typically decays into an electron, an electron neutrino and a muon neutrino.

Average half life is about 2.2μs

Experiments show that about ..... of muons, created at an altitude of 15km, reach the ground.

According to classical (non-relativistic) physics the average muon should travel:

distance = speed x time = $0.995 \times 3.0 \times 10^{8} \times 2.2 \times 10^{-6}$ = **656m**

This is not far enough for 10% to be detected at the earth's surface.

This can only be explained by applying .....

The Lorentz factor, γ corresponding to a velocity of 0.995c is 10.

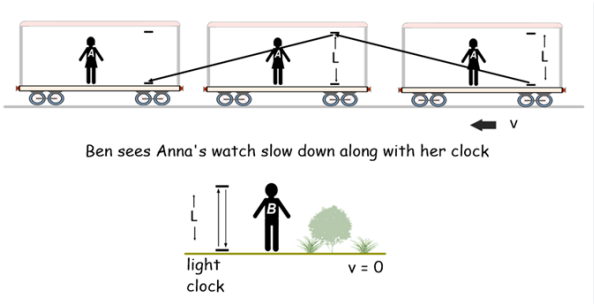
Einstein's time dilation means that for an observer in the Earth's frame of reference, the half life of muons is increased by a factor of .....

Average half life is about 2.2μs (2.2×10^{-6}s)

This means that muons actually travel 10 times further (10 x 656 = 6.56km) and so about 10% are able to reach the earth's surface.

Summary

The pulses in a light clock in a moving frame of reference have to travel further when observed from a stationary frame.



The speed of light is always the same for all observers and this means that time appears to slow down in the moving frame.

Einstein's Time Dilation Equation

Time in a moving frame appears to flow more slowly and this is called time dilation.

$$t = \gamma t_0 \qquad \text{where } \gamma = \frac{1}{\sqrt{(1 - \frac{v^2}{c^2})}}$$

Where t = the time as seen from the stationary frame of reference

t_o = the time in the moving frame (proper time)

γ = Lorentz factor

Einstein's Length Contraction Equation (Lorentz Contraction)

Length in a moving frame appears to shorten and this is called length contraction:

$$L = \frac{L_o}{\gamma} \qquad \text{where } \gamma = \frac{1}{\sqrt{(1 - \frac{v^2}{c^2})}}$$

where L = contracted length as seen from stationary frame of reference

L_o = proper length (as seen in the moving frame of reference)

v = velocity of fast moving object

c = speed of light ($3.0 \times 10^8 ms^{-1}$)

γ = Lorentz factor

Summary

There will be no change at all to lengths that are **perpendicular** to our line of motion.

The only lengths that change are those that are **parallel** to our line of motion.

Proper time and length

These are the time and length as measured by an observer who is in the **same** frame of reference.

Proper time is the time between two events which occur at the same point in space.

Proper length is the distance between two points as measured by an observer who is **at rest** with respect to these two points.

Time dilation and length contraction are really only noticeable at speeds greater than 0.5c, (approximately 1.5×10^8 ms^{-1}).

The observation of muons at the earth's surface provides direct evidence for Einstein's theory of special relativity.

Note that details of the mathematical derivation of time dilation and length contraction equations are not examinable.